ITP–SB–95–17
hep-ph/9505391
May, 1995

Universality of infrared renormalons in hadronic cross sections

Gregory P. Korchemsky* and George Sterman

Institute for Theoretical Physics,
State University of New York at Stony Brook,
Stony Brook, New York 11794 – 3840, U.S.A.

Abstract

We discuss the role of infrared renormalons in power corrections to hadronic cross sections, including their universality. We show how perturbative renormalon structure arises near kinematic boundaries in the thrust distribution, the Drell-Yan cross section and radiative B decays. The leading infrared renormalon in each case is associated with jet evolution. Demanding that the combination of perturbative and nonperturbative contributions be well-defined, we infer the form of the leading power corrections to each cross section. This is at the level $1/Q$ for the thrust distribution and Drell-Yan process, and $1/m^2$ for B decay. We discuss the universality of $1/Q$ corrections between Drell-Yan and thrust, and conclude that it is approximate, due to contributions from multijet configurations that may differ in the two cases.

to appear in the Proceedings of the XXXth Rencontres de Moriond
"QCD and High Energy Hadronic Interactions", Les Arcs, France, March 1995

*On leave from the Laboratory of Theoretical Physics, JINR, Dubna, Russia

1. Introduction. Our current understanding of hard processes in QCD is based on factorization (the "improved the parton model"), which allows us to calculate cross sections in the high energy limit $Q^2 \to \infty$ (with Q^2 a large kinematic scale) as a convolution of nonperturbative parton distribution functions and short distance partonic subprocesses. The latter are calculable in perturbative QCD as a series in $\alpha_s(Q^2)$. Although this procedure is adequate for a variety of processes, there is a wealth of data for which low-order perturbation theory alone is not enough, most notably the transverse momentum distributions of $W^\pm$ and Z^0 in the Drell–Yan process [1, 2, 3, 4] and direct photon and diphoton production [5, 6], as well as event shapes in e^+e^-–annihilation [7, 8]. As we approach the edge of the phase space in these processes, we observe a disagreement with next-to-leading perturbative QCD calculations that can be attributed to nonperturbative (or hadronization) effects, our understanding of which is still incomplete. Remarkably enough, in some cases one may achieve a better agreement with experimental data by including additional factors into parton model predictions "by hand". These factors take into account nonperturbative effects in the form of power corrections in $1/Q$. In this paper we would like to describe a simple method [9] which allows us to understand the appearence of additional nonperturbative factors in resummed cross sections. It is based on the analysis of ambiguities of the perturbative series in QCD associated with the so–called infrared renormalons [10, 11].

In the special case of processes (like the total cross section for $e^+e^- \to$ hadrons, deep-inelastic scattering, etc.), which admit the operator product expansion (OPE), nonperturbative power ("high twist") corrections to a physical distribution $\sigma(Q)$ can be parameterized by dimensional nonperturbative scales σ_n as

$$\sigma(Q) = \sigma_{\rm pert}(Q) + \sum_{n=n_0}^{\infty} \frac{\sigma_n}{Q^n}\,. \tag{1}$$

Here the first term corresponds to the perturbative contribution $\sigma_{\rm pert} = \sum_k c_k(\alpha_s(Q^2))^k$ and the leading power corrections appear as $\sim Q^{-n_0}$. While the complete sum in (1) defines a physical quantity $\sigma(Q)$, individual terms are not well defined, either due to ambiguities in the definition of nonperturbative quantities $\sigma_n \sim \Lambda^n_{\rm QCD}$ or due to divergences of $\sigma_{\rm pert}$ related to the factorial growth of the coefficients $c_k \sim \beta_0^k k!$ (with $\beta_0 = 11 - 2/3 n_f$) at higher orders of perturbative QCD. As a result, to give a meaning to the perturbative (as well as nonperturbative) contribution to $\sigma(Q)$ one has to specify the way in which we "regularize" divergences of $\sigma_{\rm pert}$. Different prescriptions lead to different results for $\sigma_{\rm pert}$ which eliminate ambiguities at the level of power corrections $\Lambda^n_{\rm QCD}/Q^n$. It is important for us that ambiguities of the perturbative contribution $\sigma_{\rm pert}$ are compensated in (1) by ambiguities in the definition of nonperturbative scales σ_n. Using this property we can explore the ambiguities of $\sigma_{\rm pert}$ to identify the structure of nonperturbative corrections, and in particular the level n_0 at which the leading power corrections appear in (1). A justification for this scheme is partly in our expectation that there is a unique theory of QCD involving perturbative and nonperturbative effects, and also that for processes in which the OPE is applicable and n_0 is known explicitly, the analysis of perturbative ambiguities leads to the same result [11].

Let us apply the same approach to hadronic processes to which the OPE is not applicable. To be concrete, we will consider the following three different processes: the thrust distribution $d\sigma/dT$ in e^+e^- annihilation, the energy distribution of lepton pairs in the Drell-Yan process and the differential rate of the radiative decay of the B meson. One of the reasons why the OPE does not apply directly to these processes is that they involve two scales Q^2 and q^2 with

$Q^2 \gg q^2 \gg \Lambda_{\rm QCD}$. (We shall see below, however, an important role for the OPE in heavy quark effective theory [12].) As a result, the short–distance partonic subprocess includes large double–logarithmic (Sudakov) corrections $(\alpha_{\rm s}(Q^2)\ln^2(Q^2/q^2))^n$, due to the enhancement of soft gluons with momenta $k \sim q$. Our strategy will be the following [9]. We first apply factorization, and perform resummation of large perturbative logarithmic corrections. The resulting resummed expressions involve integrals of the running constant at very low scales, given by soft gluon transverse momenta. This makes the perturbative series sensitive to the way in which we treat singularities of the coupling constant. Once we identify the ambiguities of the Sudakov resummed perturbative expansions, we add nonperturbative corrections in order to compensate them and make the final expressions well-defined. Although the analysis of perturbation theory cannot give us the absolute normalization of nonperturbative corrections, we may use it to parameterize nonperturbative effects by introducing new scales with dimensions fixed by the positions of infrared renormalons.

2. Sudakov resummation. For $\mathrm{e^+e^-}$ annihilation, the thrust is defined in terms of the momenta k_j of final state particles as

$$T = \max_{\vec{n}} \frac{\sum_j |\vec{k}_j \cdot \vec{n}|}{\sum_j |\vec{k}_j|}\,. \tag{2}$$

It takes a maximum value $T = 1$ for two infinitely narrow jets. For $1 - T$ small the final state consists of two jets, with invariant masses k^2 and $\bar{k}^2$, respectively, and with energy Q in the center-of-mass frame, $Q^2 \gg k^2,\ \bar{k}^2$. Then, in the $T \to 1$ limit the thrust is given by [7]

$$T = 1 - \frac{k^2}{Q^2} - \frac{\bar{k}^2}{Q^2}\,, \tag{3}$$

and for the thrust distribution one can derive the factorized form [7]

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma}{dT} \stackrel{T\simeq 1}{=} \int_0^\infty dk^2 \int_0^\infty d\bar{k}^2\ J_{\rm q}(Q^2,k^2) J_{\bar{\rm q}}(Q^2,\bar{k}^2)\ \delta\!\left(1 - T - \frac{k^2}{Q^2} - \frac{\bar{k}^2}{Q^2}\right), \tag{4}$$

where $\sigma_{\rm tot}$ is the total cross section of $\mathrm{e^+e^-} \to$ hadrons. Here, the jet distribution $J_{\rm q}(Q^2,k^2)$ describes the probability for a quark with energy Q and invariant mass $k^2 \ll Q^2$ to create a jet of collinear particles accompanied by soft gluon radiation [13, 14]. In the Born approximation, $J_{\rm q}^{(0)} = \delta(k^2)$, while the emission of soft and collinear particles by the "parent" quark leads to large logarithmic corrections $\alpha_{\rm s}^n/k^2 \ln^{2n-1} Q^2/k^2$ to $J_{\rm q}$. They can be resummed using evolution equation techniques, and the final expression for $J_{\rm q}$ can be represented to next-to-leading logarithmic accuracy in the form of a Laplace transform [13, 14, 7],

$$\int_0^\infty dk^2 {\rm e}^{-\nu k^2/Q^2} J_{\rm q}(Q^2,k^2) = \exp\left\{-\int_0^1 \frac{du}{u}\left(1-{\rm e}^{-u\nu}\right)\left[\int_{u^2Q^2}^{uQ^2} \frac{dk_t^2}{k_t^2}\Gamma_{\rm cusp}(\alpha_{\rm s}(k_t^2)) + \gamma(\alpha_{\rm s}(uQ^2))\right]\right\}. \tag{5}$$

An identical expression applies to the antiquark jet distribution. Here, ν is the transform parameter, and $\Gamma_{\rm cusp}(\alpha_{\rm s})$ and $\gamma(\alpha_{\rm s})$ are anomalous dimensions, known to the lowest orders of perturbative QCD. We notice that the jet distribution (5) satisfies the normalization condition

$$\int_0^\infty dk^2 J_{\rm q}(Q^2,k^2) = 1\,, \tag{6}$$

which guarantees that $\int dT \frac{d\sigma}{dT} = \sigma_{\rm tot}$. This means, that in agreement with the OPE for $\sigma_{\rm tot}(Q)$, large perturbative (as well as nonperturbative) corrections to the differential cross-section $d\sigma/dT$ described by (4) and (5) cancel in the total cross-section. Using (4) we find the following expression for the thrust distribution

$$\left\langle {\rm e}^{-\nu(1-T)} \right\rangle \equiv \int_0^1 dT \frac{1}{\sigma}\frac{d\sigma}{dT} {\rm e}^{-\nu(1-T)} = \left[\int_0^\infty dk^2 {\rm e}^{-\nu k^2/Q^2} J_{\rm q}(Q^2,k^2)\right]^2 . \tag{7}$$

Together with (5) this relation resums all large perturbative corrections which describe the effect of Sudakov suppression of the end–point region $T \sim 1$ in the thrust distribution $d\sigma/dT$. We notice, however, that in (5) the integration over the transverse momenta of soft gluons is potentially divergent due to singularities of the coupling constant $\alpha_{\rm s}(k_t^2)$, and it is this property of the Sudakov resummed perturbative expression which we are going to use to identify nonperturbative effects.

As a second example, we consider the distribution of the lepton pairs in the Drell-Yan process $h_1 + h_2 \to \ell^+\ell^- + X$ with respect to their invariant mass Q^2 at the edge of phase space as Q^2 approaches the invariant energy of the incoming hadrons $\sqrt{s}$. As $\tau = Q^2/s \to 1$, the final state in the partonic subprocess is dominated by soft gluons with total energy $\sim (1-\tau)Q$, which lead to the appearence of large Sudakov perturbative corrections to the short-distance partonic cross-section [13, 14, 15]. The proper quantities to study in this case are the moments of the differential cross-section $d\sigma_{\rm DY}/d\tau$, normalized to the moments of a structure function of deep-inelastic scattering $F(x)$,

$$\Delta_n = \sigma_n/F_n^2 , \qquad \sigma_n = \int_0^1 d\tau \tau^n \frac{d\sigma_{\rm DY}}{d\tau} , \quad F_n = \int_0^1 dx x^n F(x) . \tag{8}$$

Each of these quantities may be factorized into the product of moments of a parton distribution function and moments of a short-distance partonic cross-section. The parton distributions cancel in the ratio, Δ_n, which makes it possible to evaluate Δ_n perturbatively. For large n, the ratio Δ_n has large Sudakov perturbative corrections $(\alpha_{\rm s} \ln^2 n)^k$, whose origin can be traced back to the asymptotics of $d\sigma_{\rm DY}/d\tau$ and $F(x)$ as $\tau \to 1$ and $x \to 1$, respectively. In the Drell-Yan process, large corrections to $d\sigma_{\rm DY}/d\tau$ are caused by soft gluons emitted from the initial state by the quark and antiquark before they annihilate. At the same time, in deep-inelastic scattering large corrections to $F(x)$ as $x \to 1$ arise from soft gluon emission from the incoming quark, as well as from the decay of the scattered quark with energy $\sim Q$ and small invariant mass $k^2 = Q^2(1-x)/x$ into a jet of collinear and soft particles in the final state. We recognize that the latter subprocess is described by the same jet distribution $J_{\rm q}(Q^2, k^2)$ which entered into the analysis of the thrust distribution. Moreover, in the ratio of the moments Δ_n the contributions of soft gluons emitted by quarks in the initial state in the Drell-Yan process and deep-inelastic scattering cancel, and the only contribution which survives is that of the jet distribution associated with the outgoing quark in deep-inelastic scattering. As a result, for large n, $\Delta_n(Q)$ is given by

$$\begin{aligned} \Delta_n(Q) &= H(\alpha_{\rm s}(Q^2)) \left[Q^2 \int_0^1 dx x^n J_{\rm q}(Q^2, Q^2(1-x)/x)\right]^{-2} \\ &= H(\alpha_{\rm s}(Q^2)) \left[\int_0^\infty dk^2 {\rm e}^{-nk^2/Q^2} J_{\rm q}(Q^2,k^2)\right]^{-2} + \mathcal{O}(1/n) , \end{aligned} \tag{9}$$

where $H = 1 + \sum_k h_k\, \alpha_{\rm s}^k(Q^2)$ comes from the contributions of hard virtual gluons with momenta $k \sim Q$, and where the jet distribution has been defined in (5). Comparing this expression

with (7) we notice that the resummed expressions for different physical quantities, $\Delta_n(Q)$ and $\left\langle \mathrm{e}^{-n(1-T)} \right\rangle$, have the same form.

As a last example we consider the differential rate for the inclusive radiative decay $\mathrm{B} \to \gamma X_s$ in the end-point region of the photon spectrum, in the limit when the mass of the s quark is neglected. In the rest frame of the B meson, we define a scaling variable x as the ratio of the photon energy to the mass of the b quark, $x = 2E_\gamma/m$. That is, we are interested in the inclusive distribution $d\Gamma/dx$ with $x \sim 1$. In the heavy quark limit, $m \to \infty$, we may factorize the decay distribution into a convolution of the b quark distribution function in the B meson and a short distance partonic subprocess $\mathrm{b} \to \mathrm{s}\gamma$. The latter gets large perturbative corrections $\sim [\alpha_{\mathrm{s}}^n/(1-x)]\ \ln^{2n-1}(1-x)$ in the end-point region, which originate from the propagation of the s quark into the final state with a large energy $\sim m/2$ and a small invariant mass, $k^2 = m^2(1-x)$. The s quark creates a jet of collinear particles accompanied by soft radiation, which is described by the same jet distribution $J_{\mathrm{q}}(m^2, m^2(1-x))$ that we encountered above. Introducing the moments of the differential rate $d\Gamma/dx$ as

$$\mathcal{M}_n(\mathrm{B} \to \gamma X_s) = \int_0^{x_{\max}} dx\ x^n \frac{1}{\Gamma}\frac{d\Gamma}{dx}\,, \tag{10}$$

and performing the Sudakov resummation of large perturbative corrections to $\mathcal{M}_n$, for large moments n we may represent the result in the following factorized form [16]

$$\mathcal{M}_n = f_n^{(0)}\ H(\alpha_{\mathrm{s}}(m^2))\ J_n\ S_n + \mathcal{O}(1/n)\,, \tag{11}$$

where $f_n^{(0)}$ is a nonperturbative parameter describing the distribution of the b quark in the B meson, and H comes from hard gluon corrections to the effective vertex $\mathrm{b} \to \mathrm{s}\gamma$. It is similar to the analogous factor in (9). J_n takes into account all effects of the s quark fragmentation into a jet and is equal to

$$J_n = m^2 \int_0^1 dx x^n J_{\mathrm{q}}(m^2, m^2(1-x)) = \int_0^\infty dk^2 \mathrm{e}^{-nk^2/m^2} J_{\mathrm{q}}(m^2, k^2) + \mathcal{O}(1/n)\,, \tag{12}$$

with the jet distribution given by (5). There is an additional factor S_n in (11) which takes into account the nonleading contribution of the soft gluons emitted by the incoming b quark. It is defined as

$$S_n = \exp\left\{ -\int_0^1 \frac{du}{u}\left(1-\mathrm{e}^{-nu}\right)\Gamma(\alpha_{\mathrm{s}}(u^2m^2)) \right\}\,, \tag{13}$$

with $\Gamma(\alpha_{\mathrm{s}})$ an anomalous dimension [16]. Comparing (13) and (5), we notice that the contribution of S_n to the moments $\mathcal{M}_n$ is subleading with respect to that of jet subprocess, J_n, so that it can be reabsorbed into a redefinition of the anomalous dimensions Γ_{cusp} and γ to higher orders of perturbation theory.

Thus, for three processes considered above the Sudakov resummation of large perturbative corrections gives similar expressions, (7), (9) and (11). The reason for this universality is that in all cases large perturbative corrections came from the same jet subprocess which can be described by the distribution function J_{q} defined in (5).

3. Power corrections from infrared renormalons. Expressions (7), (9) and (11) were found after resummation of large Sudakov logarithmic corrections to all orders of perturbation theory. This does not guarantee, however, that the resummed perturbative series are convergent. Indeed, it is clear from (5) that the integral over the transverse momenta of soft gluons

is divergent at small values of k_t^2. In the perturbative expansion of the exponent in the r.h.s. of (5) in powers of $\alpha_s(Q^2)$ these divergences manifest themselves in factorial growth of the coefficients $\sim \beta_0^k k!$. This tells us that, taken alone, resummed perturbative predictions (7), (9) and (11) are not well defined. Since this problem arises from small momenta region of soft gluons, we do not expect perturbative QCD alone to suffice. Rather, perturbative expressions should be supplemented by nonperturbative corrections, and it is their sum that should give unique predictions for physical quantities. Although we do not know in general how to evaluate nonperturbative effects from first principles, we may explore the ambiguities of perturbative expressions like (5) to indentify potential sources of nonperturbative corrections.

Let us identify the leading infrared renormalon singularity in the expression for the jet distribution (5). It is defined by the first term in the exponent, in which we interchange the k_t and u integrals and expand the result for small transverse momentum to get [9]

$$\exp\left\{-\frac{2\nu}{Q}\int_0^Q dk_t\ \Gamma_{\rm cusp}(\alpha_{\rm s}(k_t^2))\,[1+\mathcal{O}(k_t/Q)]\right\}. \tag{14}$$

We conclude that the resummed perturbative expression for the jet distribution (5) has its leading infrared renormalon singularity at the level of a $1/Q$ power correction,

$$\int_0^\infty dk^2\ \mathrm{e}^{-\nu k^2/Q^2} J_{\rm q}(Q^2,k^2)\Big|_{\rm nonpert} \sim \exp\left(-\nu A/Q + \mathcal{O}(1/Q^2)\right), \tag{15}$$

where the dimensional parameter A includes both perturbative and nonperturbative contributions. Although the latter may depend on the process in which the jet distribution enters, the former have a universal form $\left(\sim 2\int_0^Q dk_t\ \Gamma_{\rm cusp}(\alpha_{\rm s}(k_t^2))\right)$, which follows from (14). Applying (15) to (7) and (9) we estimate the magnitudes of nonperturbative effects in the thrust distribution and in the Drell–Yan process as

$$\left\langle \mathrm{e}^{-\nu(1-T)}\right\rangle\Big|_{\rm nonpert} \sim \mathrm{e}^{-2\nu A_T/Q}, \qquad \Delta_n(Q)\Big|_{\rm nonpert} \sim \mathrm{e}^{2nA_{\rm DY}/Q}. \tag{16}$$

This analysis of perturbative ambiguities, and their relation to nonlocal operators, suggests [9] that the contribution of the leading infrared renormalon to the parameters A_T and $A_{\rm DY}$ is the same[1], although it does not allow us to draw definitive conclusions on the magnitudes of these parameters.

One may try to apply the same result to the moments of the B meson differential rate, eq.(11). There is, however, an important difference from the previous cases. For B decay, the energy scale Q, which is a kinematic invariant for the cross sections (center of mass energy for thrust and lepton mass in the Drell–Yan process), should be replaced by the b quark mass m. The $1/m$ contribution is given by an operator expectation value in heavy quark effective theory that vanishes by the heavy quark's equation of motion. This result may also be seen directly in perturbation theory. It turns out [19] that the definition of m is also ambiguous due to the presence of an ultraviolet renormalon in the perturbative series for m. Therefore, analyzing perturbative ambiguities to the moments $\mathcal{M}_n$ of the differential rate, we have to take into account contributions of infrared renormalons to the jet distribution (15) and that of the ultraviolet renormalon to the b quark mass, $m \to m+\delta m$. The net effect of the latter ambiguity

[1] This result has been recently confirmed in [17, 18]

can be reduced to a renormalization of the scaling variable $x \to x(1 - \delta m/m)$ in (10) and the renormalon contributions to the moment $\mathcal{M}_n$ can be represented in the large m limit as

$$\mathcal{M}_n\Big|_{\rm nonpert} \sim \exp(-nA/m - n\delta m/m) \tag{17}$$

with parameter A defined in (15). The absence of $1/m$ corrections implies [20] that their sum vanishes, $A+\delta m = 0$, and perturbative ambiguities will first appear in $\mathcal{M}_n$ at the level $\mathcal{O}(1/m^2)$ power corrections.

We stress that although the resummed perturbative expressions for the thrust distribution and in the B meson decay, eqs. (7) and (11), look very similar after identification of the energy Q with heavy quark mass m, their properties with respect to renormalon ambiguities are completely different, due to ambiguities in the definition of heavy quark mass m. However, keeping in mind the situation with B decay, we may interpret nonperturbative $1/Q$ corrections to the thrust distribution as a renormalization of the scaling variable $1 - T$, or equivalently the invariant masses k^2 and $\bar{k}^2$ of the jets. This implies, in particular, that the emission of nonperturbative soft gluons by the quark and antiquark makes the jets in the final state wider, $(k^2 + \bar{k}^2)/Q^2 > A_T/Q$, and the phase space for the thrust becomes smaller, $T < 1 - A_T/Q$. Indeed, using the expression (7) for the thrust distribution, and expanding it in powers of ν, one can estimate the nonperturbative correction to the average thrust from two–jet configurations,

$$\langle 1 - T \rangle \Big|_{\rm nonpert} \stackrel{2-{\rm jet}}{\sim} A_T/Q\,. \tag{18}$$

In contrast to the thrust distribution (7) at $\nu \gg 1$, the average thrust $\langle 1 - T \rangle$ gets leading contributions not only from 2–jet events but from multi–jet events as well. The latter should be analyzed separately, and there is no reason to expect that their contribution to the average thrust will be subleading by a power with respect to that in (18). Indeed, an analysis of renormalon ambiguities to perturbative series corresponding to multi–jet final state shows [21] that leading nonperturbative corrections to $\langle 1 - T \rangle$ appear at the same level $1/Q$ as in the 2–jet events, although suppressed in general by powers of $\alpha_s(Q)$. Thus, the total leading nonperturbative correction to the average thrust has the same form (18) but with the scale A_T modified by the contributions of multi–jet events. Contributions of infrared renormalons in $1/Q-$power corrections to $\langle 1 - T \rangle$ associated with multi–jet events prevent an exact equality between perturbative ambiguities in the average thrust and in the Drell–Yan process.

4. Conclusions and perspective. The analysis of ambiguities associated with infrared renormalons allows us to gain insight into the structure of nonperturbative corrections in hadronic processes to which the OPE is not applicable. Performing Sudakov resummation of large perturbative corrections, we found that in all three cases, the thrust distribution, the Drell–Yan process and the differential rate of the B meson decay, the contribution of the leading infrared renormalon has a universal form, which suggests that nonperturbative effects may appear at the level of $1/Q$ (or $1/m$ for the B meson decay) power corrections. Moreover, the $1/Q-$power corrections to the thrust distribution and to the Drell–Yan process can be parameterized by introducing two new nonperturbative scales A_T and $A_{\rm DY}$, which in an analogy with the local condensates in the QCD sum rules can be related to vacuum expectation value of nonlocal Wilson line operators [9, 21]. An important difference occurs for B meson decay, where the OPE in heavy quark effective theory ensures that infrared and ultraviolet renormalons to the

$1/m$ power corrections cancel each other, indicating that leading nonperturbative effects to the differential rate occur at the level of $1/m^2-$power corrections.

The above considerations can be easily generalized to other processes which require resummation of large perturbative corrections. Returning to the example mentioned at the very beginning, we consider the transverse momentum distribution [9] of lepton pairs in the Drell–Yan process $d^2\sigma/dq_t^2dQ^2$. In contrast with the previous case, we do not require $Q^2 \sim s$ but approach the edge of the phase space $q_t^2 \ll Q^2$. In this kinematic region, the small transverse momentum of the lepton pair is compensated by soft gluons emitted by the quark and antiquark in the partonic subprocess. These soft gluons give rise to large perturbative corrections $\alpha_s^k \ln^{2k}(Q^2/q_t^2)$, which can be resummed to all orders of perturbation theory into a Sudakov form factor [1]. As above, the resulting resummed expressions for the transerse momentum distribution suffers from infrared renormalon ambiguities which give rise to nonperturbative corrections. The leading nonperturbative corrections have a simple physical meaning [9]. They introduce an additional broading into the transverse momentum k_t-distribution of incoming quarks with a gaussian weight. In this fashion, we arrive at the form originally proposed by Collins and Soper for nonperturbative contributions to the Drell-Yan q_t–distribution [1],

$$\frac{1}{4\pi\sigma^2}\exp\left(-\frac{k_t^2}{4\sigma^2}\right), \qquad \sigma^2 = g_1 + g_2 \ln\frac{Q}{2Q_0}\,. \tag{19}$$

Here, the width σ depends on the invariant energy of the quark and antiquark in the partonic subprocess, $\hat{s} = x_1x_2s = Q^2$, and the numerical values of new nonperturbative scales (g_1, g_2, Q_0) have been estimated [4] from comparison with experiment as $g_1 = 0.15$ (GeV)^2, $g_2 = 0.40$ (GeV)^2 and $Q_0 = 2$ GeV. As we have seen, the infrared renormalons contributing to $1/Q$–power corrections have universal structure in different processes. The same is true for the infrared renormalon arising in the Sudakov resummation for the q_t-distribution. One can also show, for example, that resummation of soft gluon effects for direct photon production at small $x_F = 2q_t/\sqrt{s}$ and for diphoton production at small total q_t of the photons, leads to expressions in which the contribution of the leading infrared renormalon gives rise to nonperturbative corrections similar to those in (19).

One of us (G.P.K.) would like to thank the Organizers of the Moriond Conference for their generous hospitality and support. This work was supported in part by the National Science Foundation under grant PHY9309888.

References

[1] J.C. Collins and D.E. Soper, Nucl. Phys. B193 (1981) 381; *ibid* B197 (1982) 446. J.C. Collins, D.E. Soper and G. Sterman, Nucl. Phys. B250 (1985), 199.

[2] J. Kodaira and L. Trentadue, Phys. Lett. 112B (1982) 66; *ibid* 123B (1983) 1983.

[3] G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, Nucl. Phys. B246 (1984) 12.

[4] C.T.H. Davies and W.J. Stirling, Nucl. Phys. B244 (1984) 337;
C.T.H. Davies, B.R. Webber and W.J. Stirling, Nucl. Phys. B256 (1985) 413;
P.B. Arnold and R.P. Kauffman, Nucl. Phys. B349 (1991) 381.

[5] J. Huston et al., MSU-HEP-41027, Jan 1995, hep-ph/9501230;
A.D. Martin, W.J. Stirling and R.G. Roberts, RAL-95-021, Feb 1995, hep-ph/9502336.

[6] P. Aurenche et al., Nucl. Phys. B399 (1993) 34;
P. Chiappetta, R. Fergani and J.Ph. Guillet, CPT–94/P.3076, Dec. 1994.

[7] S. Catani, L. Trentadue, G. Turnock and B. Webber, Nucl. Phys. B407 (1993) 3.

[8] B.R. Webber, Phys. Lett. B339 (1994) 148; Cavendish-HEP-94-17, Nov 1994, hep-ph/9411384.

[9] G.P. Korchemsky and G. Sterman, Nucl. Phys. B437 (1995) 415.

[10] G. 't Hooft, *in* The Whys Of Subnuclear Physics, Erice 1977, ed. A. Zichichi (Plenum, New York, 1977), p. 943;
B. Lautrup, Phys. Lett. 69B (1977) 109;
G. Parisi, Phys. Lett. 76B (1978) 65; Nucl. Phys. B150 (1979) 163;
F. David, Nucl.Phys. B234 (1984) 237; *ibid* B263 (1986) 637.

[11] A.H. Mueller, Nucl. Phys. B250 (1985) 327; *in* QCD 20 years later, Aachen, 1992, ed. P.M.Zerwas and H.A.Kastrup, World Scientific, Singapore, 1993, v.1, p.162;
V.I. Zakharov, Nucl. Phys. B385 (1992) 452;
M. Beneke and V.I. Zakharov, Phys. Rev. Lett. 69 (1992) 2472.

[12] M. Shifman, Proceedings of the Workshop *Continuous Advances in QCD*, ed. A. Smilga World Scientific, Singapore, 1994, hep-ph/9405246.

[13] G. Sterman, Nucl. Phys. B281 (1987) 310.

[14] S. Catani and L. Trentadue, Nucl. Phys. B327 (1989) 323; *ibid* B353 (1991) 183.

[15] G.P. Korchemsky and G. Marchesini, Phys. Lett. B313 (1993) 433.

[16] G.P. Korchemsky and G. Sterman, Phys. Lett. B340 (1994) 96.

[17] Yu.L. Dokshitser and B.R. Webber, Cavendish-HEP-95-2, Mar 1995, hep-ph/9504219.

[18] R. Akhoury and V.I. Zakharov, Saclay-SPHT-95-043, Apr 1995, hep-ph/9504248.

[19] M. Beneke and V.M. Braun, Nucl. Phys. B426 (1994) 301;
I. Bigi et al., Phys. Rev. D50 (1994) 2234.

[20] A.G. Grozin and G.P. Korchemsky, OUT-4102-53, Nov. 1994, hep-ph/9411323.

[21] G.P. Korchemsky and G. Sterman, in preparation.